The Batch Pan Scheduling Problem in a White Sugar Refinery

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Overview

1. Problem Statement

2. Mathematical Model Formulation

Problem Statement

Problem Statement

- In a typical white sugar refinery, the sugar is crystallised in batch “Vacuum Pans”.
- These are evaporative crystallisers where evaporation and crystallisation take place simultaneously under vacuum.
- The vacuum is used to operate at a reduced pressure, where the boiling temperature is reduced, leading to less thermal degradation of sucrose.

Problem statement

- At the end of each batch pan cycle the pan will contain a mixture of crystals and mother liquor with 50% of the sugar (typically) being in crystal form.
- The crystallisation is followed by centrifugation to separate the crystal sugar from the mother liquor.
- Some water addition occurs at this stage to wash the sugar and dissolve small crystals that can pass into the separated mother liquor (normally called jet).
- The crystallisation is done in multiple stages with the impurities, predominantly “colour,” remaining in the mother liquor.

Aims

- Achieve the smoothest possible total steam demand from all pans
- Add waiting times between batches to achieve the required scheduling
- Do not start a batch pan unless there is sufficient liquor/jet in the feed tank to complete the batch
- There must be sufficient space in the strike receiver to accommodate the contents of a batch pan at the end of its cycle
- Processing of sugar through the centrifugals must match the requirements for correct proportional mixing of sugars of different grades (so as to maintain the required average sugar color)
- Determine the minimum number of pans necessary to meet the requirements

Crystallisation Process

The flow scheme for a typical refinery multiple stage crystallisation process is shown in the diagram below:

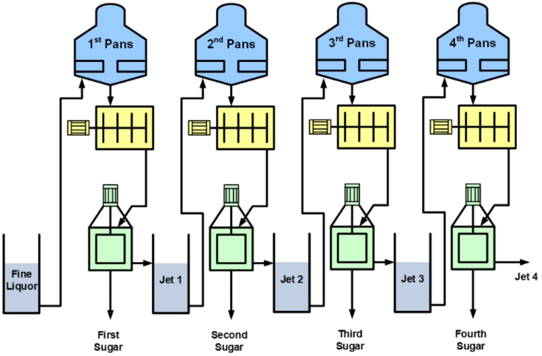


Figure 1: Multiple Stage Crystallisation Process

A batch pan cycle

A batch pan cycle consists of five main steps with different time duration:

- Step 1 (with duration t_1) consists of filling the fine liquor in the storage tank
- Step 2 (with duration t_2) consists of removing water from the liquor and feeding it to the pan, using heating steam to drive the evaporation.
- Step 3 (with duration t_3) consists of discharging the contents of the batch pan into a “strike receiver"
- Step 4 (with duration t_4) consists of processing the mixture of crystals and mother liquor in centrifugal to separate the crystals (again using steam.
- Step 5 (with duration t_5) consists of extracting the colored sugar

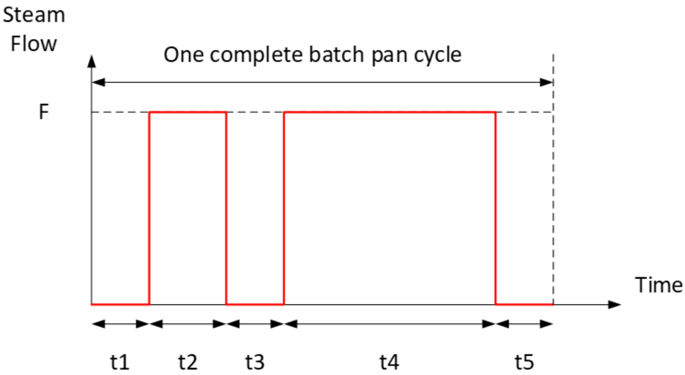
Description of a stage

A stage is considered to be the completion of a full cycle until one specific type of sugar is obtained.

- Stage 1 will correspond to the completion of a batch cycle in Pan 1 until the first sugar is extracted.
- Stage 2 will correspond to the completion of a batch cycle in Pan 2 until the second sugar is extracted. This stage will require the fine liquor from 2 cycles of Stage 1.
- Stage 3 will correspond to the completion of a batch cycle in Pan 3 until the third sugar is extracted. This stage will require the fine liquor from 2 cycles of Stage 2.
- Stage 4 will correspond to the completion of a batch cycle in Pan 4 until the fourth sugar is extracted. This stage will require the fine liquor from 2 cycles of Stage 3.

Steam flow demand for a cycle

The steam flow demand of a cycle has the same pattern regardless of the stage at which one is. The only difference between the stages is the duration of the steps within the cycles, which tend to be longer as the stages go further.



Assumed Time Data

		1st Pan	2nd Pan	3rd Pan	4th Pan
t1	minutes	12.0	12.0	12.0	12.0
t2	minutes	11.0	11.0	17.0	20.0
t3	minutes	11.0	11.0	11.0	20.0
t4	minutes	56.0	56.0	106.0	130.0
t5	minutes	10.0	10.0	10.0	10.0
total cycle time	minutes	100.0	100.0	156.0	192.0
Total steam on time	minutes	67.0	67.0	123.0	150.0
Total steam off time	minutes	33.0	33.0	33.0	42.0
Steam flow (F)	ton/h	12.4	12.4	6.7	5.5
Total steam for cycle	tons	13.8	13.8	13.8	13.8

Figure 3: The Assumed Time Data

Pre-processed possible starting times for each stage

	t_{s1}	t_{E1}	t_{s2}	t_{E2}	t_{s3}	t_{E3}	t_{s4}	t_{E4}	$t_{\dots}$
Stage 1	0	100	100	200	200	300	300	400	...
Stage 2	200	300	400	500	600	700	800	900	...
Stage 3	500	656	900	1056	1300	1456	1700	1856	...
Stage 4	1056	1248	1856	2048	2656	2848	3456	3648	...

Mathematical Model Formulation

Model formulation

The parameters:

- Let τ be the planning horizon
- Let κ be the set of the stage, $\{1, 2, 3, 4\}$
- Let τ^k be the set of all possible start time for a cycle in stage $k \in \kappa$
- Let D^k be the steam demand for a batch processing cycle in stage $k \in \kappa$
- Let C be the capacity of of a pan.

Model formulation

The decision variables:

- $X_{t^k}^k = \begin{cases} 1 & \text{if a batch cycle in stage } k \in \kappa \text{ is initiated at period } t^k \in \tau^k \\ 0 & \text{otherwise} \end{cases}$
- $V_{t^k}^k$ is the volume of liquor present in the pan at stage $k \in \kappa$ at time period $t^k \in \tau^k$.

Model formulation

The objective function:

$$\text{Min} \quad Z = \frac{1}{|\cup_{k=1}^{k} \tau^k|} \sum_{t \in \cup_{k=1}^{4} \tau^k} \left(D^k X_{t^k}^k - \frac{\sum_{k=1}^{4} \sum_{t^k \in \tau^k} D^k X_{t^k}^k}{|\cup_{k=1}^{k} \tau^k|} \right)^2 \tag{1}$$

The objective of the optimization model is to minimize the variation of steam demand over the whole planning horizon. We have modeled this as the variance of the overall steam demand.

Model formulation

The constraints Subject to:

$$V_{t^k}^{k+1} = V_{t^k-1}^{K+1} + \frac{C}{2} X_{t^k}^{k} \quad \text{for} \quad k = 1, 2, 3 \tag{2}$$

$$X_{t^{k+1}}^{k+1} \leq \frac{V_{t^k}^{k+1}}{C} \quad : \quad t^k \leq t^{k+1} \quad \text{for} \quad k = 1, 2, 3 \tag{3}$$

$$V_{t^k}^{k+1} \leq C(1 - X_{t^k}^{k+1}) \quad : \quad t^k - 1 \leq t^{k+1} \leq t^k \quad \text{for} \quad k = 1, 2, 3 \tag{4}$$

- Constraint (2) keeps track of the volume of the pan whenever a cycle in the previous stage has been initiated or not.
- Constraint (3) makes sure that it will only be possible to start a cycle in stage if the volume of liquor in that stage is full capacity.
- Constraint (4) ensure that when the cycle of a given stage is initiated, then the volume of the pan is reset to 0 in order to allow for more liquor to be put in.

Comments

- The above model is non-linear because of the objective
- It is indeed a mixed-integer nonlinear optimization problem
- Small instances of such a model can be solved using commercial solvers such as CPLEX or Gurobi.
- Some theoretical work is however needed in order to linearize the problem before one can expect to solve larger instances.
- Also heuristic or metaheuristic methods can be developed in order to find feasible solution to even larger instances.

A example of feasible schedule for three stages

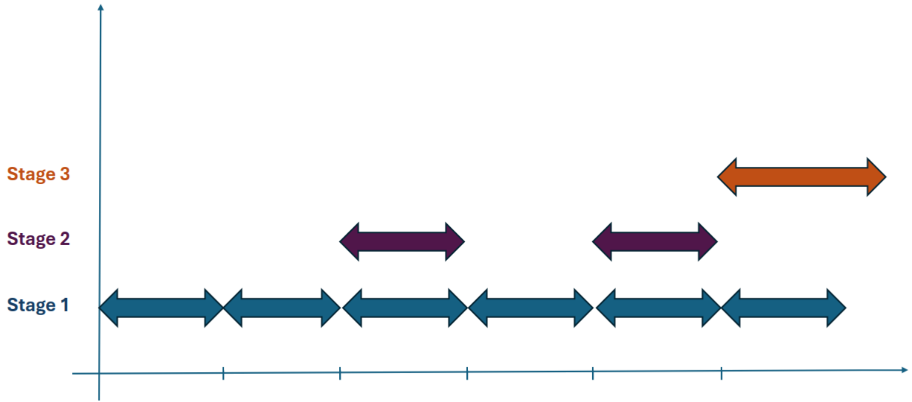


Figure 4: An example of schedule for three stages

Conclusion

- The issue is non-linear, requiring the application of appropriate linearization techniques.
- Regrettably, time constraints hindered us from conducting a thorough examination of both the first and last models.

Thank You!